Investment Analysis (part 1)—Exam 2

Paulo Maio

Spring 2013

1 Instructions

1. Maximum score is 100 points.
2. The duration is 4 hours.
3. There are 4 groups of questions with the number of points indicated in parenthesis.
4. Each group should be answered in a separate paper sheet.
5. Each paper should have your name in the top.
6. The answers should be written in a clear way!

2 Questions

2.1 Expected utility (25 points)

An individual has expected utility of the form

$$\mathrm{E}[U(W)] = \mathrm{E}(-e^{-bW})$$

ARA $\frac{U''(u)}{U'(w)} = \frac{-b^2 e^{w}}{b e^{w}}$

where $b > 0$. The individual's wealth is normally distributed as $\mathcal{N}(\overline{W}, \sigma_W^2)$. What is this individual's certainty equivalent level of wealth?

2.2 Consumption CAPM (20 points)

Let $R_{i,t+1}$ be the gross-return on an arbitrary asset i; $R_{f,t+1}$ the gross-risk free rate; and M_{t+1} the SDF in this economy. Assume that there is no conditioning information. Assume that the log return, log risk-free rate, and log SDF are given by $r_{i,t+1}$, $r_{f,t+1}$, and m_{t+1}, respectively. The log market return is denoted by $r_{m,t+1}$.

1. By assuming that the returns and the SDF are jointly log-normal, derive the expected log excess return-covariance representation of the asset pricing model associated with the SDF, M_{t+1}.

2. Now assume that the SDF has the following form:

$$M_{t+1} = \beta^{\theta} \left(\frac{C_{t+1}}{C_t} \right)^{-\frac{\theta}{\psi}} \left(\frac{1}{R_{m,t+1}} \right)^{1-\theta},$$
$$\theta \equiv \frac{1-\gamma}{1-\frac{1}{\psi}},$$

where β is a subjective discount factor; γ is the parameter of relative risk aversion; $\psi < 1$ is the elasticity of intertemporal substitution; C denotes consumption; and $R_{m,t+1}$ is the market return. Derive the expected return-beta representation (in log form) for this model. What is the sign of the consumption risk price? Interpret.

2.3 CAPM (25 points)

Consider the gross risky return for asset i, $R_{i,t+1}, i = 1, ..., N$; the gross risk-free rate, $R_{f,t+1}$; and the SDF denoted as M_{t+1}. Assume also that there is no conditioning information in this economy.

1. Derive the (unconditional) expected return-covariance representation for asset i.

2. Now assume that the SDF is given by

$$M_{t+1} = \delta R_{m,t+1}^{-\gamma}, \tag{1}$$

where δ is a time-subjective discount factor; γ is the coefficient of relative risk aversion; and $R_{m,t+1}$ is the gross market return. By using the Stein's Lemma, derive the expected return-covariance representation for asset i.

3. Interpret the sign of the risk price associated with $R_{m,t+1}$.

2.4 APT (30 points)

Consider an economy with SDF, M_{t+1}, in which the gross-return on an arbitrary asset i follows a single factor structure,

$$R_{i,t+1} = \mathrm{E}(R_{i,t+1}) + \beta_i \widetilde{f}_{t+1},$$

where $\widetilde{f}_{t+1} \equiv f_{t+1} - \mathrm{E}(f_{t+1})$ is the demeaned factor, and β_i is the regression factor beta for asset i. $R_{f,t+1}$ denotes the gross-risk free rate in this economy.

1. Derive the expected return-beta pricing equation in this economy:

   $$\mathrm{E}(R_{i,t+1}) - R_{f,t+1} = \beta_i \lambda.$$

   Derive the expression for λ.

2. Now assume that the factor is itself a return. By using the result derived in (1), derive the new expression for λ. Interpret.