

PORTFOLIO MANAGEMENT

8.1.2004

TIME: 5 h

Calculator can be used

1. You have the following information:

| | Expected return | Return standard deviation | Correlations | | |
|--------------------|-----------------|---------------------------|--------------|-----|-----|
| | | | A | B | M |
| Stock A | 12% | 30% | 1.0 | 0.4 | 0.8 |
| Stock B | 9% | 40% | 0.4 | 1.0 | 0.5 |
| Market portfolio M | 10% | 20% | 0.8 | 0.5 | 1.0 |
| Riskless asset | 5% | 0% | | | |

a.) What is the standard deviation of a portfolio where you have invested 20% in stock A, 30% in stock B, and the remaining 50% in the market index M?

b.) Calculate the betas for stocks A and B. Are the stocks correctly priced according to the CAPM, or are they over / undervalued?

c.) How much (how large a percentage) of the variance of asset A and asset B comes from unique risk and how much is due to systematic risk?

(20 p)

2. a.) Stock C and D have the same volatility, 30% per annum. The correlation between the returns on stocks C and D is 0.4. Derive analytically the weights that minimize the portfolio variance of a two-stock portfolio formed by stocks C and D.

b.) Assume that all stocks have a return standard deviation of 40%, and that the pairwise correlation between the returns on any two stocks is +0.5. Calculate the standard deviation of the return on an equally weighted portfolio consisting of a very large number of stocks.

(20p)

3. a.) Consider the following investment:

- 1.) Two years ago, you bought stocks for 100.000 mk.
- 2.) One year ago, the value of your portfolio was 115.000 mk. Then you invested 50.000 mk more into the stocks of the same company.
- 3.) Today, the value of your portfolio is 176.000 mk.

Compute the return using the formula for a.) the internal rate of return (yield, a "dollar weighted return"), and b.) geometric return (a time weighted return). Why do the results differ, and which one would you prefer in general if you worked as a fund manager for a client putting randomly money in and out of the fund?

(10 p)

PTO →

b.) Explain briefly what is meant by the following terms:

- the market model
- the Security Market Line (SML)
- the efficient frontier
- Treynor's measure

(4*2.5p)

4. Expected return on the market portfolio is 10% p.a. and its volatility is 30% p.a. The risk free rate is 3% p.a. Security X within the market portfolio has a normal market weight of 2%, a beta against the market portfolio 1.5 and volatility equal to 60% p.a.

You actively follow security X and find positive information about it which is not yet incorporated into market prices.

- a) Which expected return for security X would warrant a 10% market weight for it?
- b) Suggest an optimal portfolio for a client with constant relative risk aversion coefficient equal to 4 given that you believe in the expected return level attained in a).

(20p)

5. Short questions:

- a.) Assume that when you get older, you become **more** riskaverse. What implications would this have to your optimal total portfolio in a CAPM-world? Would different risky stocks be then selected?
- b.) What is meant by "market timing" and how could you test whether a fund manager has positive market timing ability?
- c.) Explain briefly what is meant by the "momentum strategy".
- d.) Explain briefly why Initial Public Offerings might be interesting for portfolio managers.
- e.) A risky asset has expected return of 15% and volatility 20%. The risk free rate is 3%. Suppose you can find thousands of identical risky assets with identical pairwise correlations equal to 0.2. How many, equally weighted, risky assets would it take to double the Sharpe ratio of a single risky asset portfolio?

(5 * 4 p.)

Formulas:

$$(\#1): w_m^* = \frac{E[r_m] - rf}{A \sigma_m^2}$$

$$(\#2): w_S = \frac{\sigma_B^2 - \sigma_{B,S}}{\sigma_B^2 - 2\sigma_{B,S} + \sigma_S^2} + \frac{[E_S - E_B]}{A(\sigma_B^2 - 2\sigma_{B,S} + \sigma_S^2)}$$

where E_S , E_B expected returns on stocks and bonds, and σ_S^2 , σ_B^2 , $\sigma_{B,S}$ are variances and covariance between S and B. rf is the riskfree rate. A is the coefficient of constant relative risk aversion.

$$(\#3): w_S = \frac{[E_S - rf] \sigma_B^2 - [E_B - rf] \sigma_{B,S}}{[E_S - rf] \sigma_B^2 + [E_B - rf] \sigma_S^2 - \{ (E_S - rf) + (E_B - rf) \} \sigma_{B,S}}$$

Note: $\text{Cov}(S,B) = \text{Corr}(S,B) \sigma_S \sigma_B$ and $w_B = 1 - w_S$.

(#4): 1-factor market model holds (in excess returns)

$$r_{it} = \alpha_i + rf + \beta_i (r_{mt} - rf) + e_{it}$$

Systematic risk Unsystematic risk

α_i measures the superior (abnormal) return that the active security may have! e_{it} is assumed to have mean zero and has variance of σ_{ie}^2 . Applying to #3. with $S=x$ =the active security and $B=y$ =passive benchmark:

$$\begin{aligned} E_x &= \alpha_x + rf + \beta_x (E[r_m] - rf) & E_y &= E[r_m] \\ \text{Var}(x) &= \beta_x^2 \sigma_m^2 + \sigma_{xe}^2 & \text{Var}(y) &= \sigma_m^2 \\ \text{Cov}(x,y) &= \text{Cov}(x,m) = \beta_x \sigma_m^2 \end{aligned}$$

$$w_x = \frac{\alpha_x \sigma_m^2}{\alpha_x \sigma_m^2 (1 - \beta_x) + [E[r_m] - rf] \sigma_{xe}^2}$$

and $w_y = w_m = 1 - w_x$.

$$w_x = \frac{w_0}{1 + (1 - \beta_x) w_0} \quad \text{where } w_0 = \frac{\alpha_x / \sigma_{xe}^2}{(E[r_m] - rf) / \sigma_m^2}$$

$$\#5. \beta_p^* = \frac{E[r_m] + a + b\alpha_m - rf}{A(1 - \rho^2) \sigma^2(r_m)}$$